Entry ramps in the Nagel-Schreckenberg model

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This paper describes a way of including entry ramps in the Nagel-Schreckenberg traffic model. The idea is to place what are called *shadow cars* on a highway next to cars on entry ramps, which enables the drivers to take ramp cars into account. The model is shown to capture important real-life traffic phenomena that have not been included in previous models. Furthermore, it is demonstrated that the desirable properties of the Nagel-Schreckenberg model are retained.

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I. INTRODUCTION

The Nagel-Schreckenberg (NS) model [1,2] is a traffic model based on cellular automata (see [3,4] for a description of cellular automata). It is a fairly simple model, which nevertheless has been repeatedly shown to be able to reproduce real-life traffic phenomena for highways [5,1,6,7].

Entry ramps have previously been dealt with in a very simple manner [8,9], where cars are inserted directly onto the highway with a given frequency. This method captures some important aspects of entry ramps. But there are other aspects whose modeling require more elaborate treatment. In [10] (which studies European traffic) it is shown, on the basis of empirical data, that drivers on a highway consider cars on entry ramps when choosing their own speed and lane. In countries such as Denmark this behavior is even required by law. Since these effects have not been reproduced in previous models, it means that they fail to include phenomena that have a verifiable impact on real-life traffic. Furthermore, the previous models are not very realistic when you want to simulate a larger road network, because the ramps themselves are not modeled. The goal of the work presented here is, therefore, to extend the NS model such that it incorporates the fact that drivers on highways react to cars on entry ramps. This model extension inserts objects called *shadow cars* on a highway as markers for cars on ramps. These shadow cars can then be observed by drivers on the highway, so that they can react accordingly.

The work presented here is part of an ongoing effort to expand the Nagel-Schreckenberg model and implement it in a simulation in Java (for further information on the project see [11]).

In the following four sections, the Nagel-Schreckenberg model and our extensions to it will be presented and the performance penalty incurred by the new model is examined. After the model is described we go on to verify that the model really captures the phenomena we wanted to include.

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II. THE ORIGINAL NAGEL-SCHRECKENBERG MODEL

The original Nagel-Schreckberg model was presented in [1] and has been described numerous times in other papers [12–16]. Therefore we will only briefly describe the model here.

The NS model is a cellular automata model of a single-lane road, which is divided into cells of length 7.5 m. Each cell can either be empty or contain a car. The cars have speeds that are multiples of 1 cell/time step and the length of a time step is usually 1 s, i.e., the speeds are multiples of 7.5 m/s or 27 km/h.

In each time step, the cars select a number of cells per time step, as their speed is based on their immediate surroundings. Then all of them are moved forward in parallel by the number of cells specified by their selected speed.

If we label the cars $1, \dots, n$ and use the notation in Table I, the rules for selecting speeds and moving cars can be stated as

(1) acceleration, $v_i(t) = \min(v_i(t-1)+1, v_i^{max})$;

(2) collision avoidance, $v_i(t) = \min(v_i(t), g_i^+(t))$;

(3) randomization, with probability P_{noise} let $v_i(t) = \max(v_i(t)-1, 0)$.

(4) movement, $x_i(t) = x_i(t-1) + v_i(t)$.

Rule (1) expresses the drivers' desire to be driving at the maximal speed, by accelerating at the rate of 1 cell per time step if the current speed of a car is below the maximum speed. Rule (2) makes sure a the car does not collide with another car by reducing its speed (if necessary) to the number of empty cells between the car itself and the one ahead of it. Rule (3) introduces a stochastic element that represents differing human behavior and variations in external factors, by having the car decelerate by 1 cell per time step with probability P_{noise}. Finally, rule (4) performs the actual movement of the cars.

Of course it is not enough to have rules determining how cars should be moved. Some rules regarding insertion of cars onto the road are also necessary. This is often done by inserting a new car in cell 1 with a given probability in each time step.

TABLE I. Explanation of the notation used in the basic model.

Variable	Explanation
$v_i(t)$	The speed at which car *i* is driving in time step *t*.
v_i^{max}	The maximal speed car *i* can be driving at. This is normally 3, 4, or 5 cells per second, i.e., 81 km/h, 108 km/h, or 135 km/h.
v^{max}	The highest speed of any car in the simulation. This is normally 5 cells per second, i.e., 135 km/h.
P_{noise}	The position car *i* is moved to in time step *t*. The cells are numbered 1,2, . . . in the direction of movement.
P_{noise}	A number between 0 and 1 (inclusive) that is used to introduce randomness to the speed selection.
P_{ignore}	A number between 0 and 1 (inclusive) that is used to introduce randomness to the lane selection.
$g_i^{xy}(t)$	The number of empty cells (the gap) in time step *t* between car *i* and the closest car in some direction. *x* can be *l*, *r*, or not present, *l* represents a car in the lane to the left of car *i* and *r* represents a car in the lane to the right, while a nonpresent *x* signifies a car in the same lane. *y* is either +, which means the car is ahead of car *i*, or −, which means the car is behind it. If a gap ahead of a car is larger than the car's maximum speed then the gap is simply set equal to this speed. Gaps behind the car are similarly bounded by v^{max}. When a gap refers to a nonexisting lane (e.g., g_i^{r+} for a car in lane 1) it is set to 0.
n_L	The number of lanes on the highway.
$l_i(t)$	The lane car *i* uses in time step *t*. The lanes are numbered 1,2, . . . ,n_L from right to left (relative to the direction of movement).
$d_i(t)$	The lane the driver would like to enter during time step *t*. It is called the *desired lane*.

III. LANE SELECTION RULES

When dealing with roads with more than one lane, rules are needed to let the drivers select which lane to drive in. Rules for lane selection can be divided into two categories: symmetric and asymmetric. In asymmetric rules a preference is given to a particular side of the road, in symmetric rules no such distinction is made. We have made the rules stated below (other examples of lane selection rules can be found in [12,17,18,14,6,16]), which are asymmetric, with the right side of the road being preferred for normal driving and the left part for overtaking.

(5) Consider the right lane, if $g_i^{r-}(t) \geqslant v^{max}$ and $g_i^{r+}(t) \geqslant g_i^{+}(t)$ then $d_i(t) = l_i(t-1) - 1$, otherwise $d_i(t) = l_i(t-1)$.

(6) Consider the left lane, if $g_i^{l-}(t) \geqslant v^{max}$ $[g_i^{+}(t) < \min(v_i(t)+1, v_i^{max})$ or $v_i(t) = 0]$ and $g_i^{+}(t) < g_i^{l+}(t)$ and $g_i^{r+}(t) < g_i^{l+}(t)$ then $d_i(t) = l_i(t-1) + 1$.

(7) Lane changing, with probability $1 - P_{ignore}$ let $l_i(t) = d_i(t)$ if possible. Handle the lanes one by one from right to left to avoid potential collisions.

Rule (5) represents the fact that the driver changes to the lane on the right, if moving to it will not cause the drivers in the right lane any immediate problems, and the gap ahead of him there is at least as long as that in the current lane (recall that if a gap is longer than the car's maximum speed, then it is set to the maximum speed).

Rule (6) simulates the fact that the driver should only select the lane to the left if it is "better" than the current one and that to the right, and if the driver wants to be driving faster in the next time step than the current lane allows. The check for a zero velocity is used to avoid situations where a driver is stuck in a nonmoving queue where there is room in the left lane.

The seventh rule performs lane changes and makes sure that no collisions occur (in a road with three lanes or more, it can happen that two cars both select to change to the same position of the lane between them). This rule also introduces a degree of noise in the model by making a driver disregard an otherwise desirable lane change with a probability given by the parameter P_{ignore}.

The lane selection rules presented in [12,17,18,14,6,16] are mainly rules for roads with two lanes, and many of them are of the symmetric kind. Our rules are, as stated previously, asymmetric and they directly support any number of lanes; otherwise they are similar to existing rules.

IV. THE ENTRY RAMP MODEL

Entry ramps are naturally an important part of any road network that includes highways, since they allow cars from the rest of the network to enter the highways.

Entry ramps can be modeled in different ways. In [8] an extremely simple form of entry ramps is used. This model simply ignores the entry ramps themselves and assumes that all cars from an entry ramp enter the highway in a certain cell, and in each time step a new car (driving at a relatively low speed) is inserted here with a given probability. This model is so simple that it is extremely efficient seen from a computational point of view, and as reported in [8] it is able to reproduce phenomena from real traffic. In [9] a similar approach is used, except that each entry ramp is represented by more than one cell on the highway.

On the other hand it can be argued that if the focus of interest is the entry ramps themselves, or if it is desired to simulate a larger network, then the simple model is not detailed enough. One concrete problem is that (as mentioned in the Introduction) empirical studies (for example, [10]) have shown that in European traffic, drivers on a highway notice cars driving on entry ramps and often make room allowing them to enter more easily (in countries such as Denmark this behavior is actually required by law). Behavior such as this is of course not captured by the simple entry ramp model.

To capture such behavior *shadow cars* are placed on the highway parallel to cars on entry ramps (an example of shadow car insertion is shown in Fig. 1). A priority is assigned to each shadow car based on how close the associated car is to the end of the entry ramp. This priority is used as the probability that a car on the highway takes the car on the ramp into account.

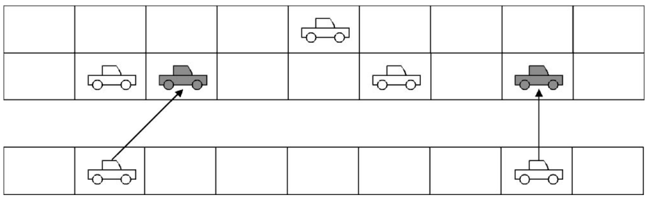

FIG. 1. Illustration of placement of shadow cars on the highway. The upper lattice is a highway and the lower one is an on-ramp, which has not connected with the highway yet. The gray cars are shadow cars, which are observed by the drivers on the highway allowing them to take cars on the ramp into account when deciding what to do.

Each car on the highway has a flag $f_i(t)$ called the *comply flag*, which has the value $true$ if the car currently takes cars on ramps into consideration. Cars whose comply flag is set to true are said to be in the *complying mode*, and if it is *false* they are said to be in the *neutral mode*. Associated with this flag is a counter $c_i(t)$, which is set to 3 every time car i takes a ramp car into consideration. $c_i(t)$ is decreased by one for every iteration where car i does not see any shadow cars. When $c_i(t)=0$ the comply flag $f_i(t)$ is set to false. Cars in complying mode will take cars on the entry ramp into consideration regardless of priorities, while cars in neutral mode only do so with a probability given by the priority of the shadow car. This system is used to avoid erratic behavior.

Furthermore, it should be noted that cars driving on a ramp have their maximum speed reduced by 1 cell/s.

The notation used in the entry ramp rules is explained in Table II. The notation defined in Table I is still valid, though the gaps now include shadow cars on equal footing with the other cars, if the car itself is in complying mode.

The new rules for speed selection are (1) acceleration, $v_i(t)=\min(v_i(t-1)+1,v_i^{max})$; (2) collision avoidance, $v_i(t)=\min(v_i(t),g_i^{nsc+}(t))$; (3) shadow car compliance, if $f_i(t)$ and $v_i(t)>v_i^{comply}$ and $v_i(t)>g_i^{sc+}(t)$ then let $v_i(t)=\lfloor[v_i(t)+g_i^{sc+}(t)]/2\rfloor$; (4) randomization, with probability P_{noise} let $v_i(t)=\max(v_i(t)-1,0)$; (5) movement, $x_i(t)=x_i(t-1)+v_i(t)$.

Every time a gap length is computed for car i and a shadow car j is found, $f_i(t)$ is set to true with probability $p_j^{sc}(t)$, that is the shadow car will be taken into consideration with a probability that increases toward the end of the ramp.

The first rule is equivalent to the original first rule. The second rule looks a bit different now because it directly specifies that shadow cars are not taken into account. In the third rule, however, it is tested whether the car should comply with an observed shadow car (if any) ahead of it. The criteria here is, as mentioned previously, that the car should be in the complying mode [$f_i(t)$ should be true], and also that the car should not be driving below a certain speed v_i^{comply}. This later criterion constitutes a lower limit of how slow a car on a highway will drive to make room for a car on a ramp. If the criteria is fulfilled and the gap up to the shadow car is smaller than the current speed of the car, then its speed is reduced to the average of the gap length and the current speed of the car. This rule make cars on a highway slow down if there are cars on an entry ramp, but it avoids abrupt breaking. Finally the fourth and fifth rules are equivalent to the third and fourth rules of the original NS model.

TABLE II. Explanation of the notation used for the entry ramp rules. Please note that all the gaps defined in Table I now include shadow cars if the car is in the complying mode.

Variable	Explanation
l_{ramp}	The length of a ramp (it should be clear in each case as to which ramp is meant).
$p_i^{sc}(t)$	The priority of the shadow car associated with car i in time step t.
$f_i(t)$	The *comply flag* of car i at time step t. If it is set to *true* all shadow cars are taken into consideration no matter what, otherwise the priority of the shadow cars are used.
$c_i(t)$	The number of time steps (after time step t) until car i enters the neutral mode.
$g_i^{sc+}(t)$	The same as $g_i^+(t)$ except that it is the gap up to the next shadow car.
$g_i^{nsc+}(t)$	The same as $g_i^+(t)$ except that it is the gap up to the next nonshadow car.
$x_i^h(t)$	The rightmost cell on the highway that is next to the cell of car i, which is driving on an entry ramp in time step t.
$g_i^{h+}(t)$	The number of empty cells (the gap) in time step t between cell $x_i^h(t)$ and the closest car ahead of it in the rightmost lane of the highway.
$g_i^{h-}(t)$	The number of empty cells (the gap) in time step t between cell $x_i^h(t)$ and the closest car behind it in the rightmost lane of the highway.
$v_i^{h-}(t)$	The speed of the car that gave rise to the value of $g_i^{h-}(t)$ in time step t.
v_i^{comply}	At speeds lower than this, drivers on the highway will not reduce their speed to make room for cars on an entry ramp (usually 4 cell/s).
$w_i^+(t)$	A measure of how well the highway can accommodate car i if it enters the highway during time step t.
$w_i^-(t)$	A measure of how much it will inconvenience the car on the highway behind car i if car i enters the highway during time step t.

For each of the cars on an entry ramp the following rules are applied to manage the associated shadow cars: (6) shadow car, move the shadow car associated with car i to the first (if any) of the following cells which are not in use,

$$x_i^h(t), x_i^h(t)+1,\ x_i^h(t)-1, x_i^h(t)-2, \ldots,\ x_i^h(t)-v^{max} \quad (7)$$

compute priority $p_i^{sc}(t)=x_i(t)/l_{ramp}$.

Rule 6 places a shadow car (if possible) for the ramp car. The first priority is to insert the shadow car parallel to the car on the ramp. If this is not possible then the next cell (in the direction of movement) is tried. One could argue that since cars on entry ramps on average have a lower maximum speed than cars on a highway, it would be a good idea to place the shadow cars behind the car on the ramp. This however can lead to a very bad situation where a car on a highway is parallel to a car on a ramp and the two cars are driving at the same speed. In this situation the car on the highway would never notice the car on the ramp if it was not attempted to place the shadow car ahead of the car on the highway. If both these cells are occupied, then the attempt to notify the car parallel to the ramp car is abandoned, and it is instead attempted to place the shadow car further to the back. In such a situation it probably would not make sense to attempt to insert shadow cars further ahead, since the car in cell $x_i^h(t)$ will anyway attempt to slow down or change lane because of the car ahead of it. Furthermore, there would be no benefit from making cars on the highway ahead of the car on the ramp slow down, because of a shadow car inserted much ahead of the ramp car, since this would risk making the problems worse for the ramp car.

In rule 7 the priority of the shadow car is computed as the fraction of the length of the ramp, which the car on the ramp has traveled. This makes sure that it becomes increasingly likely that the shadow car is taken into consideration as the car itself reaches the end of the ramp and, therefore, has an increasing need of entering the highway.

The lane selection rules for cars on highways are not changed directly (though the rule numbers should now be 8, 9, and 10), but it should be noted that if a car enters (or is in) the complying mode, then shadow cars will be treated as ordinary cars.

Entry ramps are assumed to have only one lane and they are divided into two parts. The first part does not allow cars to enter the highway, while the second part does. Because of the single-lane assumption cars driving on entry ramps do not use the normal lane selection rules. Instead, the cars driving on the second part of the ramp use the following rules to decide whether or not they want to enter the highway:

(8) inconvenience factor. $w_i^-(t)=\max(v_i^{h-}(t)-g_i^{h-}(t),1)$.

(9) accommodation factor, $w_i^+(t)=\max(v_i(t)-g_i^{h+}(t),1)$;

(10) highway entry, with probability $p_i^{sc}(t)/w_i^-(t)w_i^+(t)$, let $x_i(t)=x_i^h(t)$ and $l_i(t)=0$.

The inconvenience factor $w_i^-(t)$ computed in rule 8 is a measure of how well car i can enter the highway without inconveniencing the car behind it—a low number signifies little inconvenience. The accommodation factor $w_i^+(t)$ computed in rule 9 describes how well the area on the highway ahead of car i can accommodate a car driving at car i's speed—a low number indicates good accommodation. The final rule is used to decide whether the driver actually wants to enter the highway or not. The probability of this is the priority of the associated shadow car divided by the inconvenience and accommodation factors. This means that the more suitable the situation on the highway, the more likely it is that the driver will choose to enter the highway, and that the closer a car is to the end of a ramp, the more likely it is that its driver will ignore an unfavorable situation and enter anyway.

Note that collisions could occur if a car decides to enter the highway in the same time step in which another car decides to change to the rightmost lane of the highway. This is avoided by handling the lane changes on the highway first and then cars entering from the ramps.

V. PERFORMANCE

The rules we have added to handle entry ramps make the model slightly more complicated, in this connection the following issues should be considered.

(1) The number of cars on the ramps will normally be insignificant in comparison with the number of cars on the highway, so any increase in computational costs incurred only for cars on the ramps are not very important.

(2) Extra computational costs that are only incurred close to ramps are not very significant since ramps normally only cover a small part of a highway.

(3) The cars on the ramps that have to follow the fresh rules do not perform the normal lane selection rules and it is only the cars on the second part of the ramps that need to perform the three rules for entry selection.

(4) The handling of the comply flag and the associated counter cost a bit of extra work for each car.

(5) The gap computations for all cars with the fresh rules have to distinguish between shadow cars and normal cars, but it is easy to implement it in such a way that most of the additional work is only performed if a shadow car is encountered, which only happens close to the ramps.

(6) Rule 3 increases the computation time spent on each car slightly. But it can easily be implemented in such a way that the only extra cost in areas where there are no ramps is for a single check of the comply flag.

These issues show that there will be a performance penalty when using the proposed entry ramp rules, but the penalty will be relatively small.

It would have been nice to measure the performance impact of using the fresh rules (relative to using the previous rules). This, however, would require identical simulations with the fresh and the previous rules, and that cannot be done since the two sets of rules would lead to different events occurring. The result of this is that it would be hard to discern whether the increase in running time was caused by the execution of the rules or by the effects the rules have on the traffic.

The (small) performance penalty we expect can be further decreased by only applying the fresh rules in the vicinity of the ramps. One could, for instance, in each time step examine the area of a highway just before each ramp and for each car in that area swap the object/function that implements the previous rules with an object/function that implements the fresh rules. Similarly the object/function implementing the previous rules could be reinstated in each car after each ramp. The result of this would be that the performance overhead of the fresh rules would only be paid in the places where there actually are ramps, and this (as mentioned above) tends to be a rather small part of the total highway length.

The fresh rules also use more memory than the previous rules because the addition of the $f_i(t)$ flag (which is a Boolean) and the counter $c_i(t)$ (which is normally a number between 0 and 3) is required for each car. This additional information takes up anywhere between 3 bits and 8 bytes depending on how it is encoded. The previous model requires, at the very least, the storage of current speed $v_i(t)$, maximum speed v_i^{max}, and the desired lane $d_i(t)$. Since the speeds are normally between 0 and 5 they can be squeezed into 3 bits each and the desired lane could be stored as -1, 0, or $+1$ (denoting the change in lane number relative to the current lane number), which requires 2 bits. This means that the required information takes up at least 8 bits and normally it would be encoded in three integers, which takes up 12 bytes.

The above discussions show that the increase in memory use is between 38% and 67%, for each car. But it should be noted that the road structure will normally take up more space than the cars themselves, since each cell (whether empty or occupied) will normally be a 4-byte reference or a 8-bit state regardless of which rules are used, and there will normally be many more cells than cars in the system.

It should also be noted that it often will be a good idea implementationwise to store information about where each car is (i.e., road segment, lane, and cell index) in the cars themselves, since this will lower the number of parameters that have to be passed around. This increases the memory requirements for both the previous and the fresh rules by the same amount thereby decreasing the relative increase in memory use for the fresh rules.

VI. SIMULATION RESULTS

The results of this paper are divided into two parts: first it is verified that drivers on the highway react to shadow cars, and then it is shown that the model retains the desirable properties of the NS model. The first part contains no references or comparisons with other NS ramp models because to the best of our knowledge there are no descriptions in the literature of models that display any of the effects we look for.

The verification of the models was performed by implementing it on a computer. The simulations contained a single highway with a number of entry ramps each 50 cells long, and the last 20 of these allowed entry to the highway.

A. Influence of cars on an entry ramp on drivers on the highway

We have tested whether the model captures the effect on drivers on the highway who consider cars on the entry ramp when choosing a lane. This was done by performing 16 000 time steps of a two-lane highway of 2×300 cells with an entry ramp connected to the highway at cell 200.

Densities were measured as the number of cars in cells 185–199 (inclusive) on the highway divided by 15. Note that observations that did not include any cars were discarded.

To eliminate variations (other than those we intended to measure) P_{noise} was set to zero and v_i^{max} was set to 5 cells/s for all i. Cars were inserted on the highway with fixed probabilities of 0.25 for the right lane and 0.15 for the left lane. Cars were also inserted on the ramp, but these cars remained where they were placed. At every 1000th time step, one of these stationary cars was inserted in one of the last 15 cells of the first part (cells 16–30) of the ramp. The setup for the simulation is illustrated in Fig. 2.

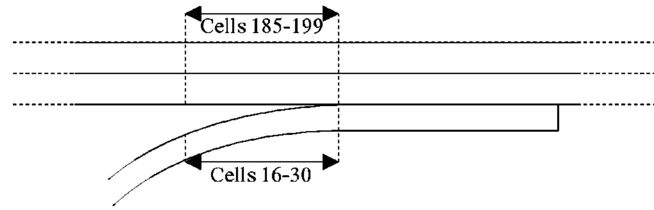

FIG. 2. Illustration of where the measurements were made in the first experiment. Stationary cars were inserted in cells 16–30 on the entry ramp and car densities were measured in cells 185–199 on both the lanes of the highway.

Not allowing the cars on the ramp to enter the highway is of course a bit artificial but it ensures that there are no queues started by cars entering the highway propagating back and thereby influencing the measurements.

If the model has the desired properties we should be able to see an increase in the number of cars in the left lane of the highway compared to the total number of cars (we denote this relationship by $d_{l/r}$) when the ramp density increases.

In Fig. 3, $d_{l/r}$ is shown as a function of the number of cars on the ramp and it is seen that $d_{l/r}$ indeed does increase with increasing ramp density.

To show that highway drivers decrease their speed in response to cars on an entry ramp, a simulation similar to that described above was performed. In this simulation the highway had only one lane, because it makes sure that the only possible adaption was a speed decrease. The insertion probability was 0.2. Since the drivers cannot change lanes to accommodate cars on the ramp, we will expect the entry ramp to act as a flow-conserving bottleneck for the traffic on the highway. Actually the ramp in the previous two-lane situation is also a kind of flow-conserving bottleneck, because it certainly is a bottleneck and the flow is conserved—however it differs from, e.g., a tunnel, because the right lanes gets an

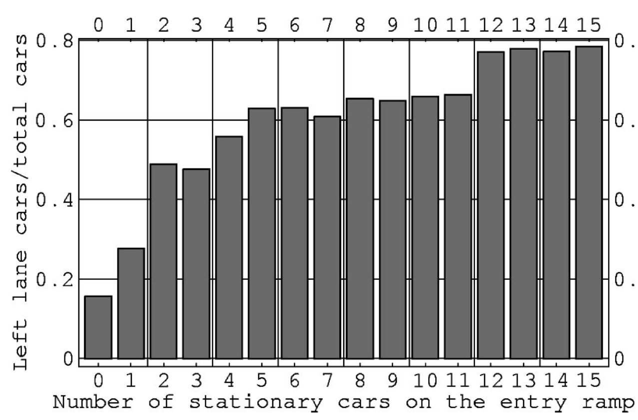

FIG. 3. The average value of $d_{l/r}$ as a function of the number of cars on the ramp. Data in which all the observed cells on the highway were empty were discarded.

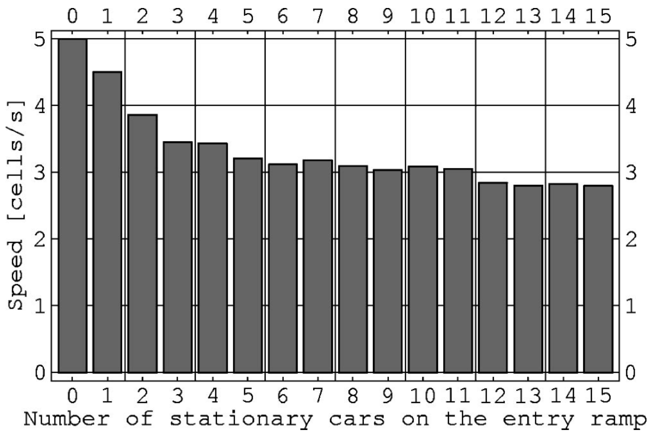

FIG. 4. The average speed of cars on the highway as a function of the number of cars on the ramp. Data in which all the observed cells on the highway were empty were discarded.

unusual role in that it is allowed but "undesirable" to use it.

Figure 4 shows a plot of the speed of the cars on the highway as a function of the number of cars on the ramp. From this plot we conclude that the ramps do in fact act as a bottleneck, i.e., the drivers lower their speed when they observe cars on an entry ramp as desired.

The two simulation runs described above are artificial in nature because they were meant to focus on one specific phenomenon and cut away all sources of noise. Apart from those tests we have also looked for the same effects in less artificial situations. This was done by performing a simulation of a two-lane highway of 2×600 cells with a single entry ramp connected to the highway at cell 300. Cars were inserted on the highway with probabilities 0.3 and 0.2 for the right and left lanes, respectively, and 0.25 for the ramp, P_{noise} was set to 0.2, P_{ignore} was set to 0.01, v_i^{max} was 3 cells/s with 10% chance, 4 cells/s with 60% chance and 5 cells/s with 30% chance and data were averaged over 15 cells on each side of each cell. The cars inserted on the ramp were inserted at the beginning of the ramp, they moved like normal cars (though their maximum speed were reduced by one cell per second) and they were allowed to enter the highway.

We are again interested in seeing whether drivers adjust their behavior to accommodate cars on the entry ramp. Therefore it is still undesirable to have jam clusters created by cars entering the highway propagate back to the segment before the ramp is connected to the highway, because this will make it hard to tell whether changes in the traffic patterns are due to the jam clusters or due to the behavior we want. Because of this the insertion probabilities have been chosen so low that jam clusters should be rare—on the other hand we do not want to select the insertion probabilities so low that a car on the highway seldom encounters a shadow car.

The results of this simulation is shown in Figs. 5 and 6, which show $d_{l/r}$ and average speed, respectively, as they develop through space and time.

A further plot was made from this simulation (Fig. 7). It shows the density of cars during the simulation. This figure illustrates that hardly any queuing took place, which means

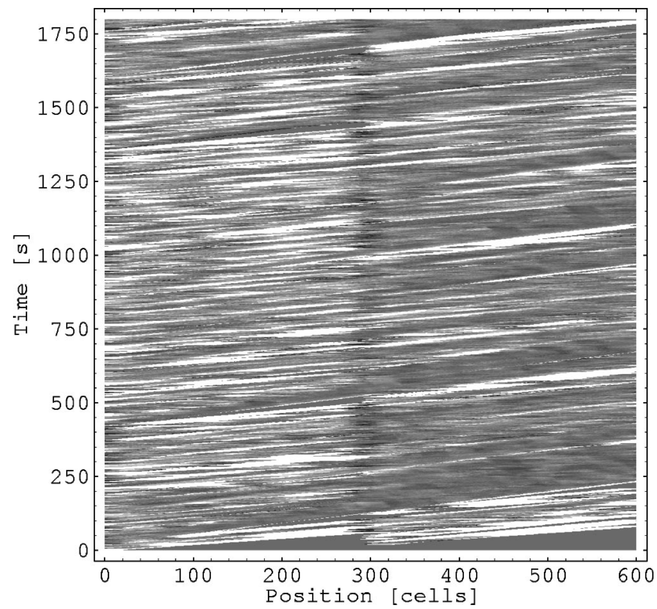

FIG. 5. The fraction of cars on the highway that are in the left lane ($d_{l/r}$) as a function of space (the abscissa axis) and time (the ordinate axis). Dark areas signify a high fraction of cars on the left lane while light areas are areas with the most cars in the right lane.

that lane changes and speed reductions are not caused by queues.

Figure 5 clearly illustrates that numerous cars change to the left lane and Fig. 6 shows that the drivers actually do slow down when they pass the ramp. It is seen that both of these effects are observed *before* the place where cars from the entry ramp are allowed to enter the highway—this is something that previous versions of the NS model have been

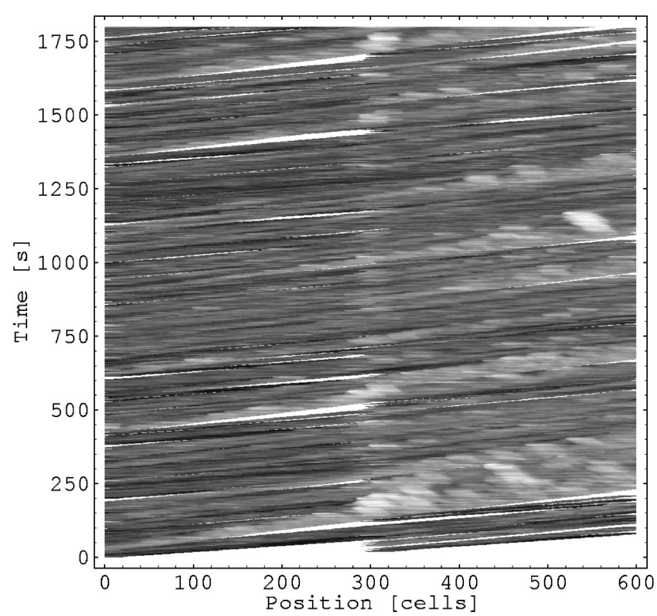

FIG. 6. The average speed of cars on the highway as a function of space (the abscissa axis) and time (the ordinate axis). Dark shades represent high speeds and lighter shades represent low speeds.

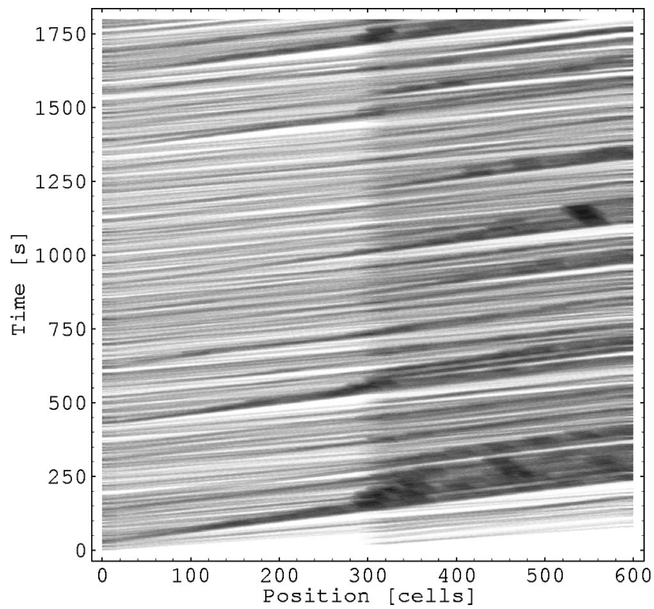

FIG. 7. The density of cars on the highway as a function of space (the abscissa axis) and time (the ordinate axis). The shading represents the density (the darker it is the more cars there are).

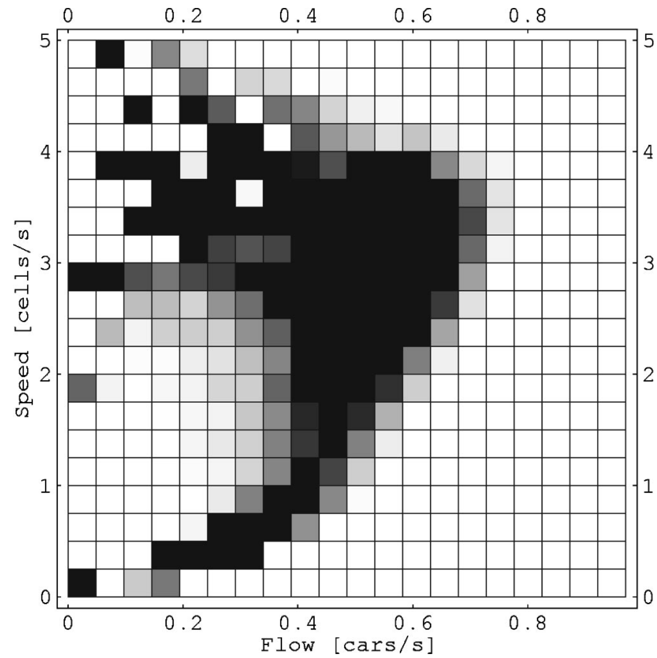

FIG. 8. The average speed (along the abscissa axis) as a function of flow (along the ordinate axis). Speed is measured in cells/s and flow in cars/s. The color indicates the number of observations, with black representing 10 000 or more observations and white representing close to zero observations. Data for this figure were collected for all the cells of the highway.

unable to include even though they are relevant for real-life traffic [10]. The long white patches in the left half of Fig. 6 are caused by a group of fast cars leaving a group of slow cars behind, thereby generating an empty region between them that grows in length.

Finally it should be noted that Fig. 7 shows that we seem to have struck a nice balance between the desire not to create jam clusters and still have enough traffic to be able to observe something interesting.

B. General verification of the model

To verify that the model (just as other variations of the NS model [1,6,7] is a reasonable description of real-life traffic, plots of the average speed, flow, and density as functions of each other were made and compared to their real-life counterparts. These diagrams are expected to show nothing that has not been produced by previous models, since the effects we are looking for are of a more local nature than what can be seen from such diagrams.

To make these plots a simulation run was performed that contained a two-lane highway 1600 cells long and two entry ramps. The first ramp was connected to the highway at cell 200 and the other at cell 900, and the cars on the entry ramps were (as in the previous experiment, but contrary to the first two) normal cars that were inserted at the beginning of the ramp, were driving normally (though with a maximum speed that were reduced by 1 cell/s), and were allowed to enter the highway.

For each cell on the highway and for each time step the density and speed were recorded. This was done by looking at 15 cells on each side of the given cell and in both lanes. This means that the density recorded for a given cell was the number of cars within 15 cells on either side of the cell itself in both lanes divided by 62, and the speed recorded was the

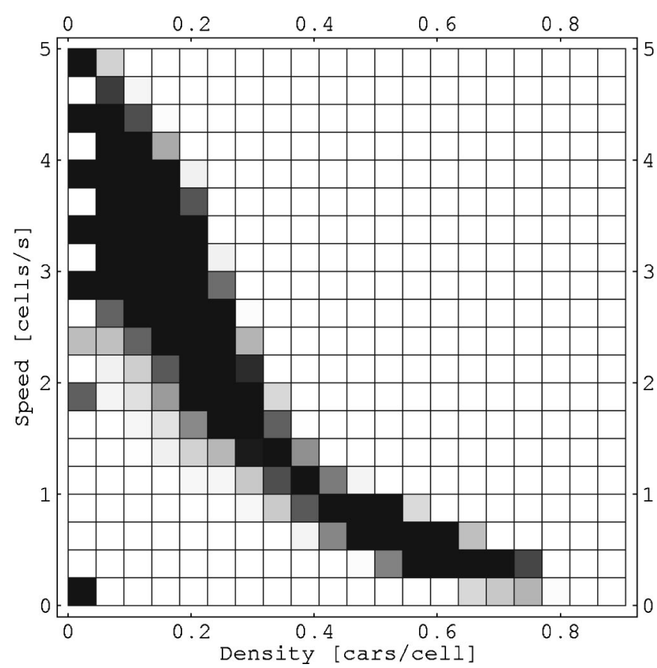

FIG. 9. The average speed (along the abscissa axis) as a function of density (along the ordinate axis). Speed is measured in cells/s and density in cars/cell. The color indicates the number of observations, with black representing 10 000 or more observations and white representing close to zero observations. Data for this figure were collected for all the cells of the highway.

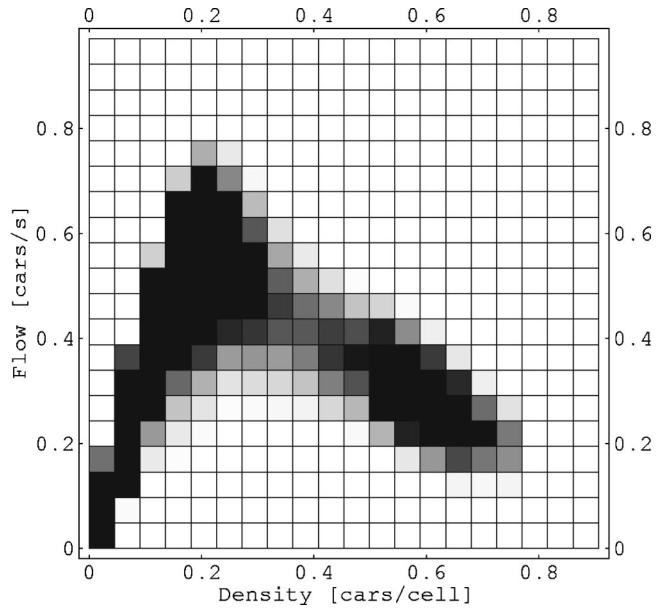

FIG. 10. The flow (along the abscissa axis) as a function of density (along the ordinate axis). Flow is measured in cars/s and density in cars/cell. The color indicates the number of observations, with black representing 10 000 or more observations and white representing close to zero observations. Data for this figure were collected for all the cells of the highway.

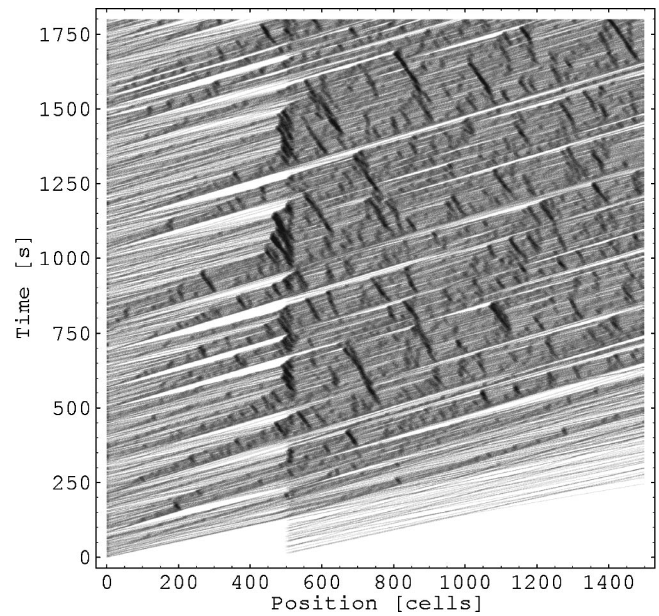

FIG. 11. The density on the highway as a function of space (the abscissa axis) and time (the ordinate axis). Black corresponds to high densities and white to low densities.

average of the speeds of the cars within that area. The flow through a cell was computed as the density multiplied by the average speed. Figures 8, 9, and 10 are based on these kinds of measurements for all the cells of the highway.

The roads were initially empty and cars were inserted in the first cell of each lane of the roads with a given probability, which increased from 0 to 1 in steps of 0.01 for every 20 time steps. Each time a car was inserted, a value of v_i^{max} was selected for it. With 10% chance 3 cells/s were selected, 4 cells/s with 60% chance, and 5 cells/s with 30% chance.

In Fig. 8 speed is plotted as a function of flow, and it is seen that the plot is similar to the plots in [6,7], which are based on real-life traffic counts. In the leftmost part of the top half of the diagram it is seen that there have been many observations of the speeds 3, 4, and 5 cell/s at low flows. This represents the fact that the road initially was empty and the insertion probability was initially very low. Therefore several cars driving alone at their own maximum speed (which could be either 3, 4, or 5 cells/s) were observed. Moving further to the right of the diagram it is seen that as the flow increases the average speed drops slightly, until the traffic becomes congested and the speed and flow drops dramatically.

Figure 9 shows average speed as a function of density. It shows that at low flow values, cars drive at their maximum speed, and as the density increases, the speed drops very fast and the maximum speeds of the cars become unimportant because they are impeded by the traffic. When the density gets sufficiently high it is seen that the drop in speed slows down.

Comparing Fig. 9 to the corresponding real-life plot in [16], it is seen that they have the same overall shape, however the spread of the real-life observations are smaller, but that is most likely because the real-life data are 1-min averages.

A plot of flow as a function of density can be found in Fig. 10. It clearly shows that a lot of the traffic is divided into two parts. In the first one, flow increases linearly with density because there is room enough for everyone to be driving at their maximum speed. However in the second part, the flow decreases with increasing density because the traffic becomes congested. The shape of the plot is the same as in the corresponding plot based on real-life data in [16], but the real-life plot has a larger spread in the second part.

Another plot was also made that displays the density of cars on the highway as a function of time and space. The plot was expected to display waves of certain densities traveling forwards and backwards on the highway.

The simulation giving rise to the density plot was a simulation of a two-lane highway 1500 cells long. It included a single entry ramp connected to the highway at cell 500.

The data was only averaged over five cells on each side of each cell. The reason for this is that for the plots of speed, density, and flow as functions of each other, a larger granularity is desirable to make the plots useful, while for the density time-space plot it is the local fluctuations that are interesting, so it is not desirable to smoothen the measurements out too much.

The density plot is shown in Fig. 11. It exhibits the expected waves of high density traveling slowly backwards through the highway (the so-called shock waves) and waves of low density traveling forward fast. Furthermore, we see that, as mentioned in [8], the ramp serves as a nucleation point for high densities. This is also seen in real traffic.

VII. CONCLUSION

We have successfully extended the NS model with an entry ramp model that captures the effect of drivers on the highway adjusting their behavior to make room for cars on entry ramps, a phenomenon that is observed in real-life traffic [10], but has not been included in previous models. Figures 3–7 clearly demonstrate that drivers in our model actually do modify their speed and lane to make room for cars driving on an entry ramp *before* the cars on the ramp enter the highway. Figures 8–10, furthermore, show that while the model now has a locally more correct behavior around the entry ramps it still retains the desired overall relationships between flow, speed, and density. Finally Fig. 11 illustrates that the model still produces waves of different densities traveling forward and backward.

The extra functionality of the model comes at a relatively small computational cost.

In the future we hope to get more detailed real-life data that will enable a better verification and calibration of the model.

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[1] K. Nagel and M. Schreckenberg, J. Phys. I **2**, 2221 (1992).
[2] M. Schreckenberg, A. Schadschneider, K. Nagel, and N. Ito, Phys. Rev. E **51**, 2939 (1995).
[3] M. Gardner, Sci. Am. **223** (10), 120 (1970).
[4] S. Wolfram, *Cellular Automata and Complexity* (Addison-Wesley, Reading, MA, 1994).
[5] D. Helbing and M. Schreckenberg, Phys. Rev. E **59**, 2505 (1999).
[6] K. Nagel, D. E. Wolf, P. Wagner, and P. Simon, Phys. Rev. E **58**, 1425 (1998).
[7] M. Rickert, P. Wagner, and Ch. Gawron, in *Proceedings of the Fourth PASA Workshop, 1996*, edited by F. Hoßfeld, E. Maehle, and E.W. Mayr (World Scientific Publishing, Singapore, 1996).
[8] E. G. Campari and G. Levi, Eur. Phys. J. B **17**, 159 (2000).
[9] G. Diedrich, L. Santen, A. Schadschneider, and J. Zittartz, Int. J. Mod. Phys. C **11**, 335 (2000).
[10] M. Aa. Nielsen and R. Rysgaard, Danish Road Directory Technical Report No. 27, 1995 (unpublished).
[11] J. Holm, M. M. Pedersen, and P. T. Ruhoff, in *Proceedings of Trafikdage på Aalborg Universitet 2001*, edited by A. Lohmann-Hansen and J. Nielsen (Aalborg University Press, Aalborg, 2001) 2001), Vol. 2, pp. 397–405.
[12] K. Nagel *et al.*, Los Alamos National Laboratory, Technical Report No. LA-UR 97-3531, 1997 (unpublished).
[13] D. Huang and Y. Wu, Phys. Rev. E **63**, 022301 (2001).
[14] W. Knospe, L. Santen, A. Schadschneider, and M. Schreckenberg, Physica A **265**, 614 (1999).
[15] L. Roters, S. Lbeck, and K. D. Usadel, Phys. Rev. E **59**, 2672 (1999).
[16] P. Wagner, K. Nagel, and D. E. Wolf, Physica A **234**, 687 (1997).
[17] M. E. Foulavand, Phys. Rev. E **62**, 5940 (2000).
[18] M. Rickert, K. Nagel, M. Schreckenberg, and A. Latour, Physica A **231**, 534 (1996).